6-D Cooling using a Super-FOFO Channel

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6-D Linear Theory

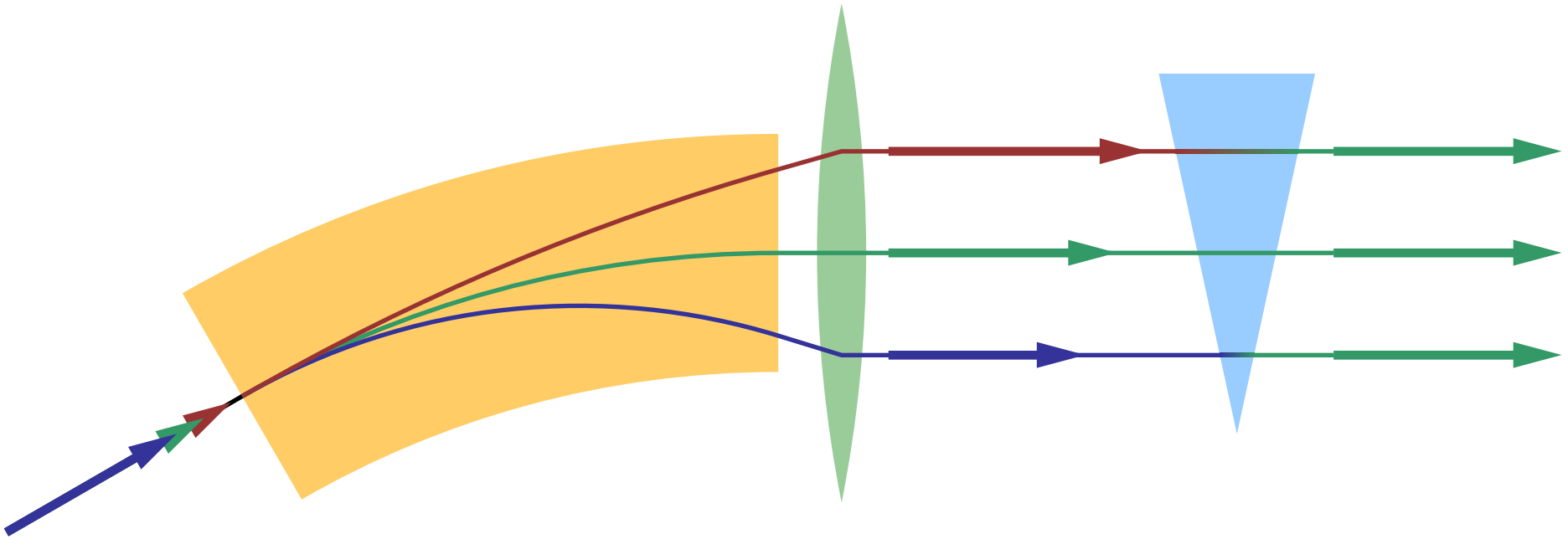
- Lattice cell described by linear matrix M
- Matrix has eigenvalues

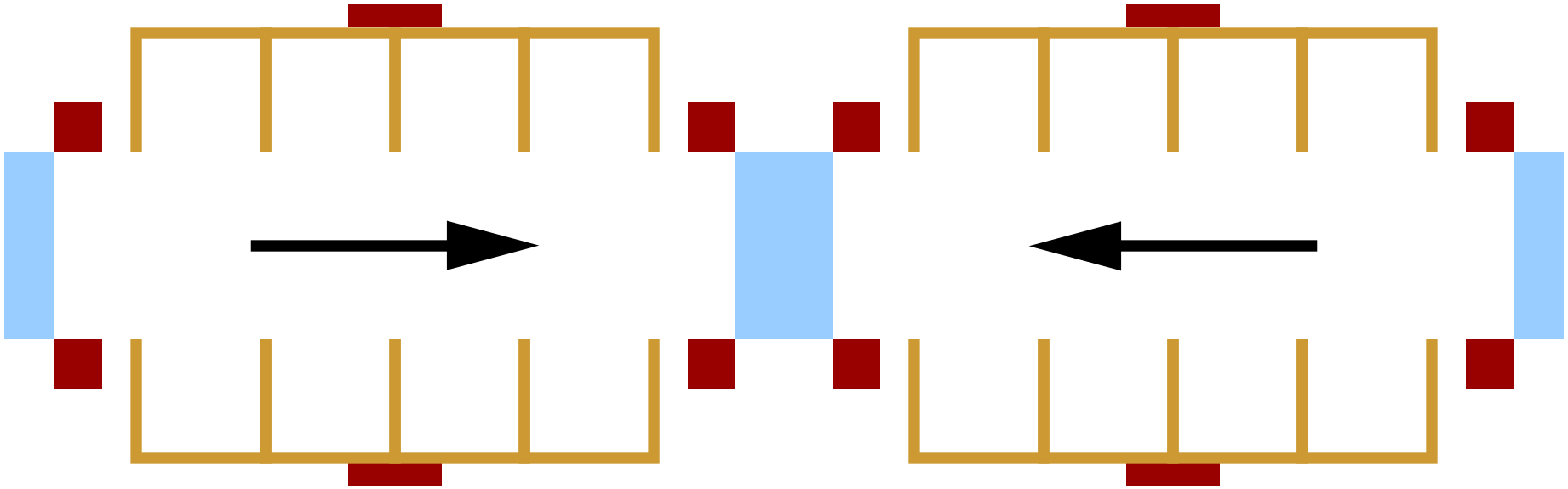
$$Mv_\alpha = \lambda_\alpha v_\alpha$$

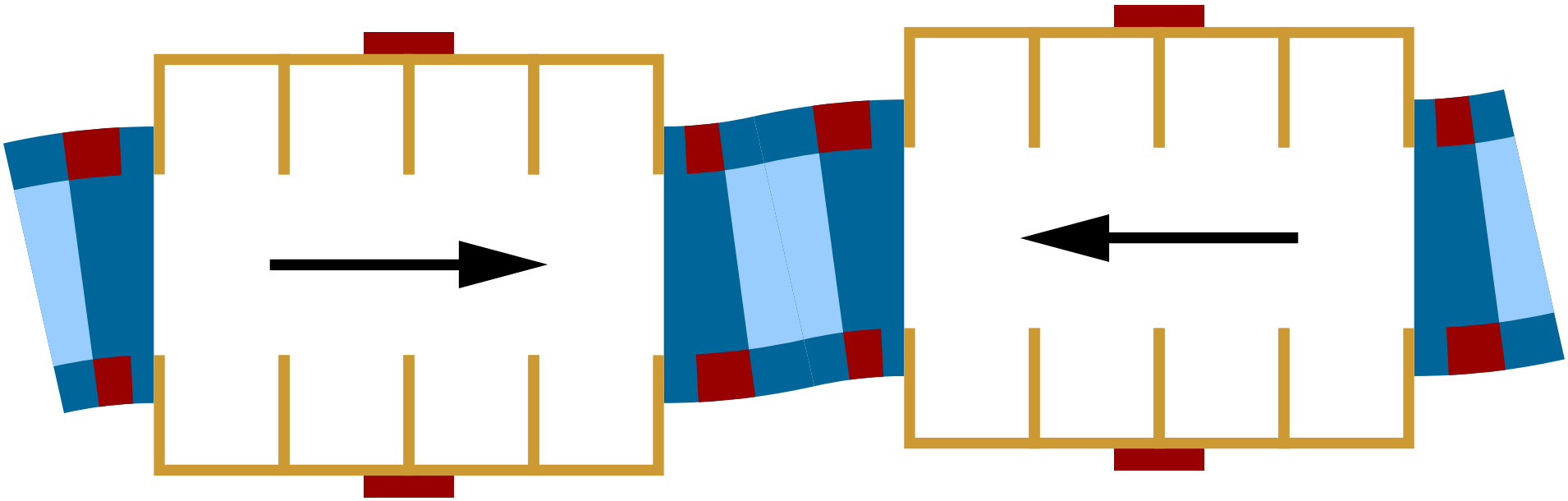
- Without absorbers, stable when $|\lambda_\alpha| = 1$
- With absorbers
  - Straight lattice: four $|\lambda_\alpha| < 1$ (transverse), two $|\lambda_\alpha| > 1$ (longitudinal)
  - General lattice
    - Cooling in all planes: all $|\lambda_\alpha| < 1$
    - Product of eigenvalues depends only on absorber lengths along reference orbit
    - Decrease in longitudinal $|\lambda_\alpha|$ requires increase in transverse $|\lambda_\alpha|$

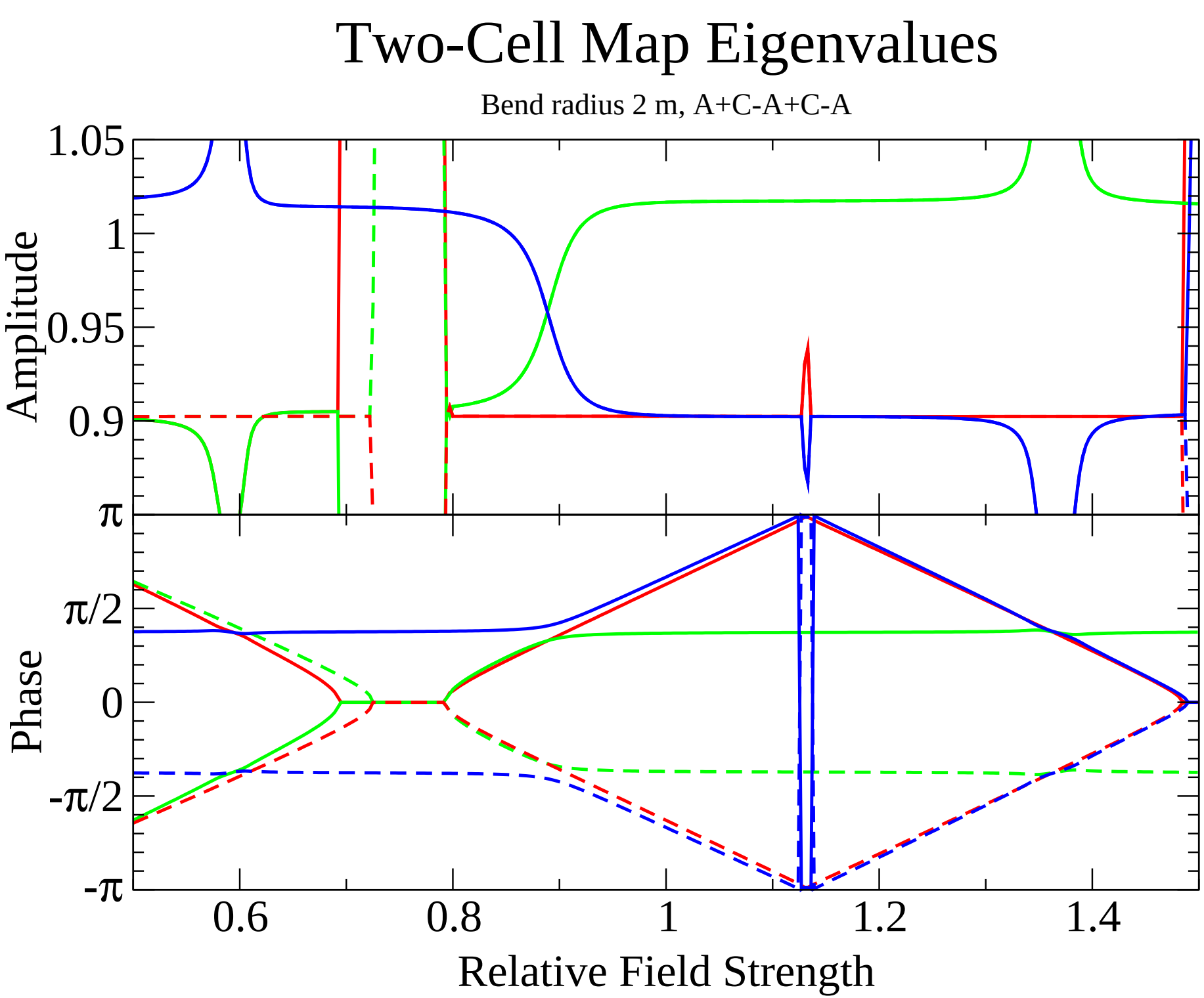
Achieving Eigenvalue Mixing

- Must couple longitudinal to transverse
- Traditional: create dispersion at wedge
- Rotate ellipse in 6-D phase space
  - Rotate by having dispersion in RF cavities
  - All three phase ellipses have projection in transverse momentum direction
  - Transverse momentum projection decreased
  - Rotation small except near synchro-betatron resonance
    - ⋆ Limited energy bandwidth?
- Angle on absorber faces
  - Need dispersion at absorber: position depends on energy
  - Energy reduction dependent on position: coupling
  - Non-symplectic: no need to be near resonance for coupling



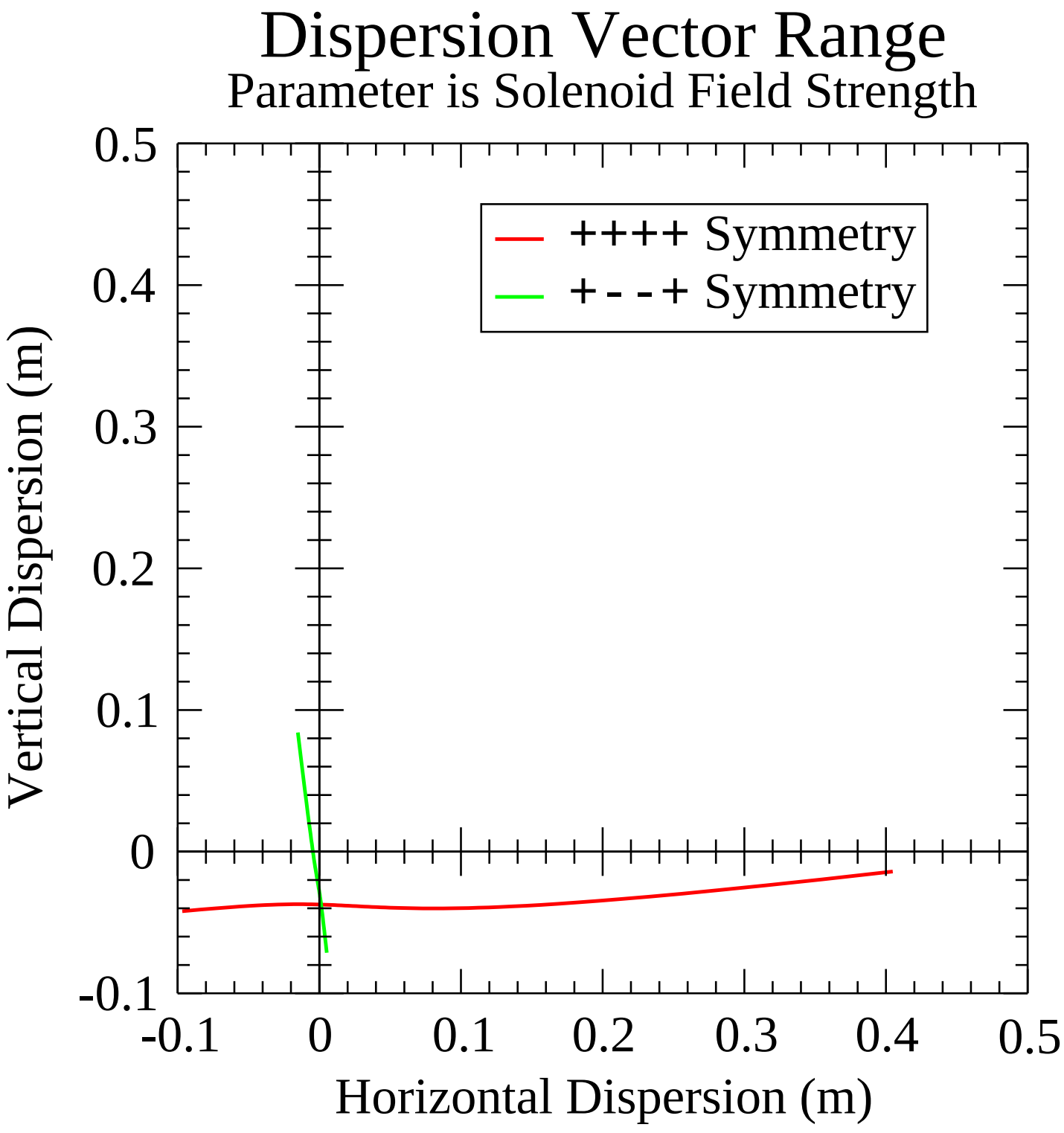


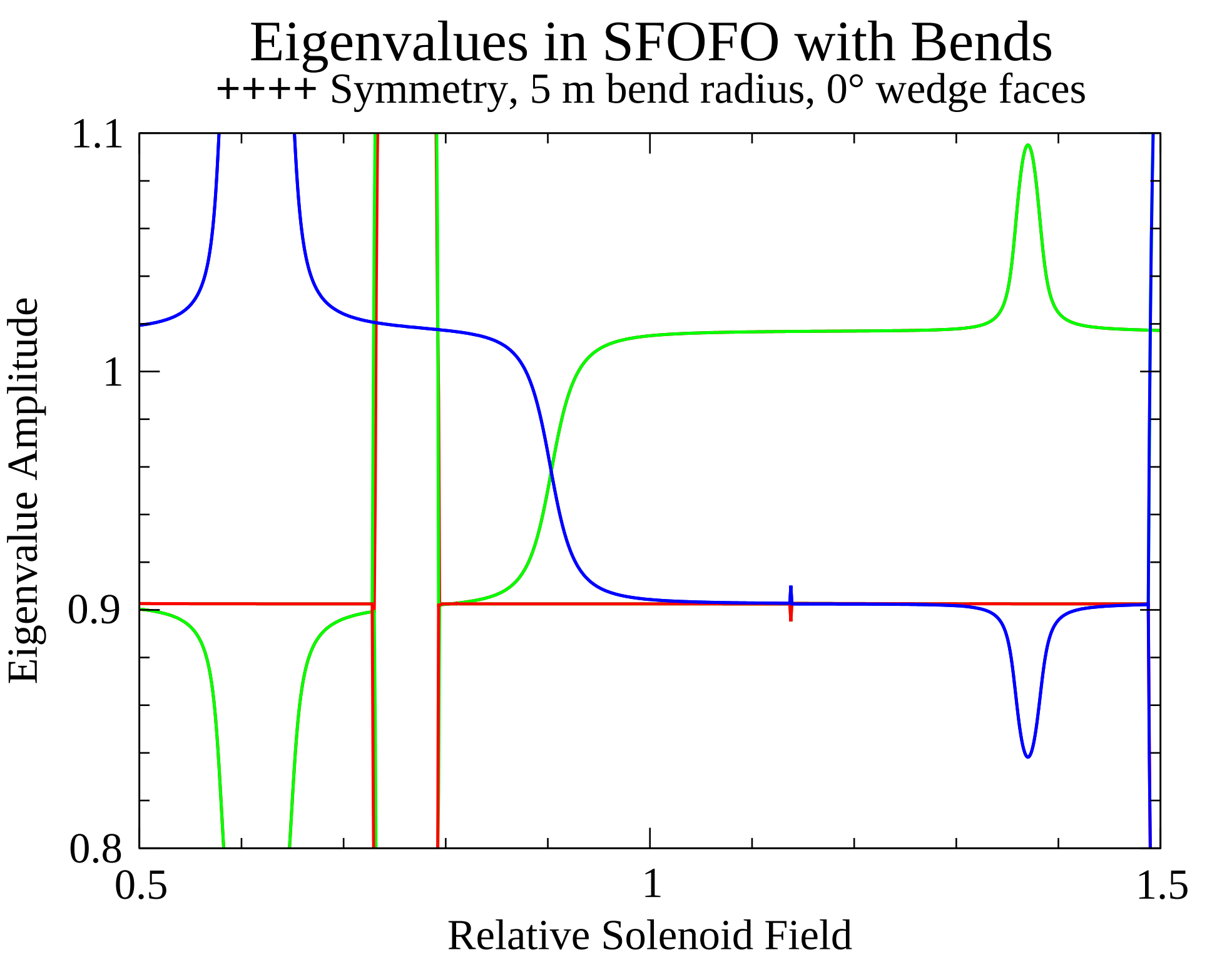


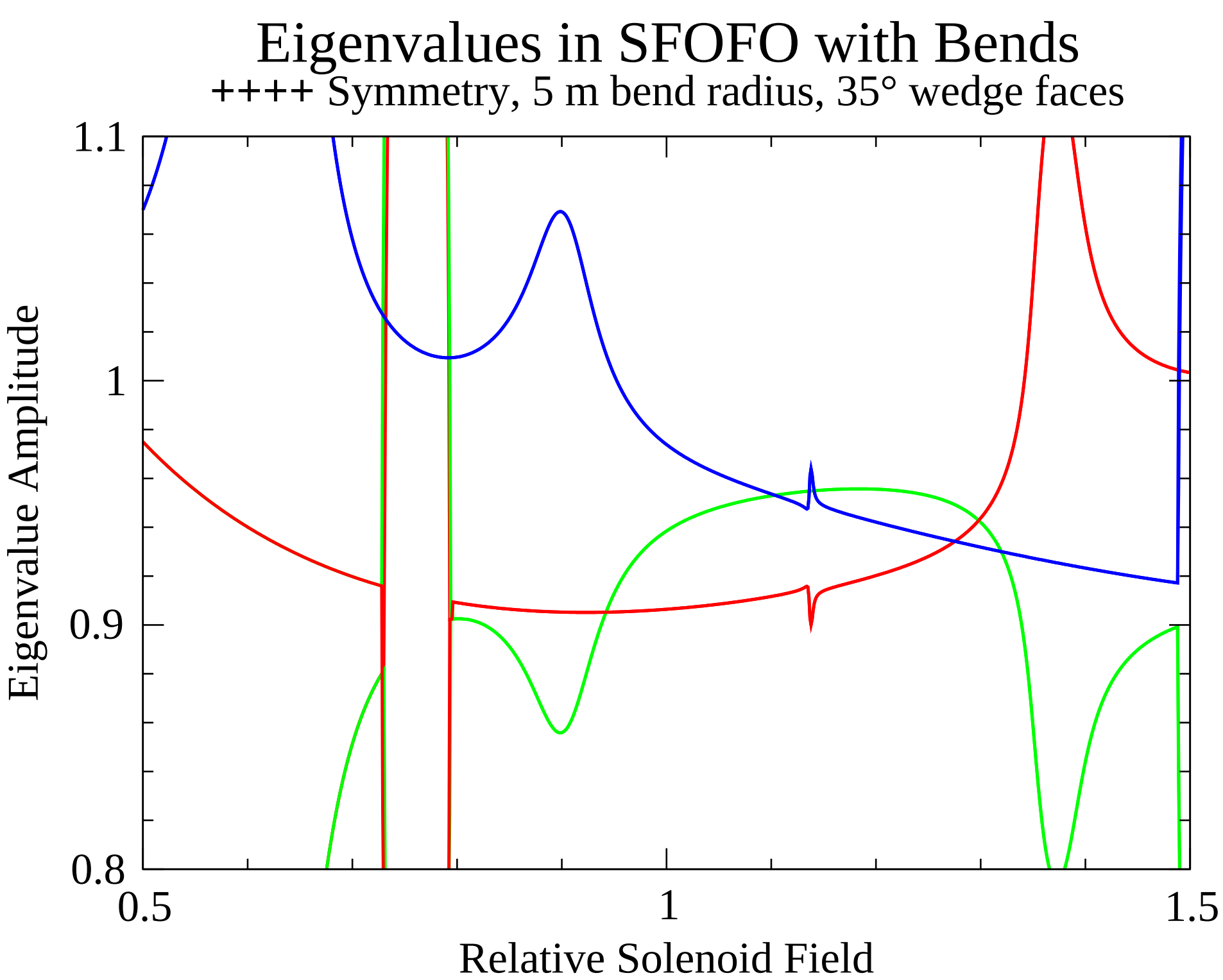

Two-Cell Map Eigenvalues
Bend radius 2 m, A+C-A+C-A
Amplitude
1.05
1
0.95
0.9
Phase
π
π/2
0
-π/2
-π
0.6
0.8
1
1.2
1.4
Relative Field Strength

Results with Angled Faces

- Bend symmetries must be chosen properly
  - Some symmetries give transverse momentum dispersion, not position dispersion (+-+-, ++--)
  - +--+ symmetry gives no average dispersion
- Get wider bandwidth, flatter eigenvalues than resonance method
- $\pi/2$ resonance of two-cell system: small effect
  - Nonlinear trouble?
  - Smaller dispersion, more face angle fixes


Dispersion Vector Range
Parameter is Solenoid Field Strength
++++ Symmetry
+--+ Symmetry
Vertical Dispersion (m)
Horizontal Dispersion (m)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.1
0
0.1
0.2
0.3
0.4
0.5


Eigenvalues in SFOFO with Bends
++++ Symmetry, 5 m bend radius, 0° wedge faces
Eigenvalue Amplitude
1.1
1
0.9
0.8
0.5
1
1.5
Relative Solenoid Field


Eigenvalues in SFOFO with Bends
++++ Symmetry, 5 m bend radius, 35° wedge faces
Eigenvalue Amplitude
1.1
1
0.9
0.8
0.5
1
1.5
Relative Solenoid Field